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FRACTURE & COMPLEXITY

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ABSTRACT

The lecture deals with the opposite natural trends in composite systems: catastrophe and chaos arising from simple nonlinear rules, as well as order and structure emerging from heterogeneity and randomness.

Part I deals with *Nonlinear Fracture Mechanics* models (in particular, the *Cohesive Crack Model* to describe strain-localization both in tension and in compression) and their peculiar consequences: *fold catastrophes* (post-peak strain-softening and snap-through instabilities) or *cusp catastrophes (*snap-back instabilities) in plain or reinforced structural elements. How can a relatively simple nonlinear constitutive law, which is scale-independent, generate a size-scale dependent ductile-to-brittle transition? Constant reference is made to Dimensional Analysis and to the definition of suitable nondimensional *brittleness numbers* that govern the transition. These numbers can be defined in different ways, according to the selected theoretical model. The simplest way is that of directly comparing critical LEFM conditions and plastic limit analysis results. This is an equivalent way --although more effective for finite-sized cracked plates-- to describe the ductile-to-brittle size-scale transition, if compared to the traditional evaluation of the crack tip plastic-zone extension in an infinite plate. In extremely brittle cases, the plastic zone or process zone tends to disappear and the cusp catastrophe conditions prevail over the strain-softening ones and tend to coincide with the LEFM critical conditions in the case of initially cracked plates.

Part II deals with the occurrence of self-similar and fractal patterns in the deformation, damage, fracture, and fragmentation of heterogeneous disordered materials, and with the consequent apparent scaling in the nominal mechanical properties of the same materials. Such a scaling is negative (lacunar fractality) for tensile strength and fatigue limit, whereas it is positive (invasive fractality) for fracture energy, fracture toughness, and fatigue threshold. At the same time, corresponding fractal (or renormalized) quantities emerge, which are the true scale-invariant properties of the material. They appear to be the constant factor (the universal property) in the power-law relating the nominal canonical quantity to the size-scale of observation. When the reference sets from self-similar become

self-affine, we obtain *Multi-fractal Scaling Laws,* which are asymptotic and present a decreasing fractality for increasing structural sizes. They reproduce the experimental data very consistently. On the other hand, *Critical Phenomena* are always associated to the emergence of self-similar or self-affine patterns, to fractal (renormalized) or multi-fractal quantities, and to spontaneous self-organization. Typical examples are represented by: phase transformations, laminar-to-turbulent fluid flow transitions, avalanches in granular media, earthquakes, micro-cracking and fracture in structural materials. In a fractal framework, it is then possible to define a scale-invariant constitutive law: the so-called *Fractal Cohesive Crack Model*, in which stress and strain are defined over lacunar fractal sets and the fracture energy in an invasive fractal set, which is the Cartesian product of the two previous sets.

REFERENCE

A. Carpinteri, *Fracture and Complexity*, Springer-Nature, Heidelberg, 2021.